Determination of a Galaxy Luminosity Function

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Introduction

A luminosity function for galaxies gives the number of galaxies or the number density of galaxies as a function of luminosity. Since luminosity and absolute magnitude are related only through the solar values of these quantities, the luminosity function can equivalently be expressed as a function of absolute magnitude. Luminosity functions are typically determined for samples of galaxies in a particular region of space, such as clusters.

The function can take many forms. A general analytical form for a differential luminosity function that seems applicable to a wide range of samples of galaxies was proposed by Paul Schechter.

The Schechter luminosity function is expressed as:

\[
\phi(L) \, dL = \phi^* \left( \frac{L}{L^*} \right)^\alpha \exp \left( - \frac{L}{L^*} \right) \frac{dL}{L^*}
\]

where \(\phi(L)dL\) is the number of galaxies that have luminosities between \(L\) and \(L + dL\) per unit volume of space.

Rewritten as a function of absolute magnitude, the luminosity function becomes:

\[
\phi(M) \, dM = \frac{2}{5} \phi^* (\ln 10) \left[ 10^{\frac{2}{5}(M^* - M)} \right]^\alpha \exp \left[ -10^{\frac{2}{5}(M^* - M)} \right] dM
\]

where \(\phi(M)dM\) is the number of galaxies that have absolute magnitudes between \(M\) and \(M + dM\) per unit volume of space.
In either expression, \( L^* \) (or \( M^* \)), \( \alpha \), and \( \phi^* \) are parameters used to obtain the best fit to the data. Given a plot of the number of galaxies in a luminosity interval versus luminosity interval, \( L^* \) is a characteristic luminosity at which the slope of the function changes rapidly. If absolute magnitude is used instead of luminosity, the definition of \( M^* \) is completely analogous. The parameter \( \alpha \) is the slope of a plot of \( \log \phi \) versus \( \log L \) (for \( L < L^* \)), and \( \phi^* \) is a normalization parameter (usually expressed in units of \( \text{Mpc}^{-3} \)).

A luminosity function for a given sample of galaxies is specified by these parameters. However, the values of the parameters differ from one sample of galaxies to another. For example, \( \alpha = -1.24 \pm 0.02 \) for the Virgo cluster, but it is consistent with \(-1.0\) for the local group of galaxies near the Milky Way. Therefore, a universal luminosity function does not exist. Instead, each luminosity function depends on the region in which the sample of galaxies is found.

By determining the luminosity function for a given sample of galaxies, several properties of that sample can be investigated. Luminosity functions for galaxies have shown that the number of galaxies falls off rapidly as the luminosity increases. This behavior indicates that the number of low-luminosity galaxies far exceeds the number of high-luminosity galaxies in any given sample. Assuming the sample consists of galaxies in a cluster, the unobserved dwarf galaxy populations can be determined. In addition, the mass distributions in clusters, and possibly, the distances to clusters are measurable through the information contained in the luminosity function for the region (cluster).

**Method**

The luminosity function for a particular cluster or sample of galaxies can be determined by counting the number of galaxies in specific apparent magnitude ranges. Assuming the distance to each galaxy in the sample is known the measured apparent magnitudes can be converted to absolute magnitudes through the distance modulus equation. A fit to the plot of the number of galaxies in an absolute magnitude interval versus absolute magnitude interval gives the luminosity function. This approach is particularly effective for clusters since all the galaxies within the cluster lie at approximately the same distance.

A method for determining the luminosity function of a sample of galaxies where the distance to relatively few of the galaxies is known was proposed by S. Phillipps and T. Shanks in the *Monthly Notices of the Royal Astronomical Society*. The method involves measuring the excess of correlated galaxies close to a galaxy of known distance as a function of magnitude.

Their sample consisted of galaxies of known redshift (known distance) and a much larger number of galaxies of known apparent magnitude. For each redshift galaxy, the number of galaxies within a circle of specified radius centered on the redshift galaxy was counted as a function of magnitude. The galaxy number information was distributed into appropriate magnitude bins.

Due to the clustering of galaxies, there will be an excess of galaxies above the expected background count around the redshift galaxy. The background count can be calculated for the sample as a function of apparent magnitude.
The excess will be a result of the galaxies that are physically associated with the redshift galaxy. Therefore, the excess galaxies lie at the same distance as the redshift galaxy. With the number of galaxies, apparent magnitude, and distance data, one can determine the luminosity function for this sample.

This method was evaluated to determine if it could be applied to a sample of more distant galaxies. If the method appears feasible, then it will be used to measure the luminosity function for this sample.

**The Data**

The sample of galaxies to which this method of determining the luminosity function could be applied consists of 491 objects. Redshifts have been measured for 151 of the objects in this sample. The total number of galaxies in the sample is actually less than the total number of objects, since several stars are included in the field. The redshift and flux (magnitude) measurements that were performed on the galaxies were carried out on the stars in the sample as well. Therefore, any analysis of the data must reject the number and magnitude contributions from the stars in order to ascertain a valid galaxy luminosity function.

Galaxies with redshifts of $z \sim 0.5$ (approximately 100 in the field) will be considered when applying the method of Phillipps and Shanks to this sample. This redshift corresponds to an average distance of 1500 Mpc. Therefore, this sample will indeed be deeper than the one considered by Phillipps and Shanks. Those galaxies with redshifts that are either much greater than or much less than 0.5 are excluded from the analysis because of the poor statistics associated with such objects when using this method on the sample.

**Evaluation of Method**

Several theoretical calculations were carried out in order to determine if using the galaxy correlation technique was feasible for a deeper sample. The initial calculation ascertained the number of background and correlated galaxies that would be expected to be observed in the circle centered on the redshift galaxy. From these values, a signal-to-noise ratio was determined. The number of galaxies in a given volume of space can be calculated by integrating the equation:

$$dN = N_0 \left[ 1 + \left( \frac{r_0}{r} \right)^\gamma \right] dv$$

where $N_0$ is the mean number density of galaxies in the volume, $\gamma$ and $r_0$ (in units of Mpc) are constants, and $r$ is the distance from the center (redshift) galaxy to a correlated galaxy. The $N_0dv$ term gives the number of galaxies that would be found in the volume $dv$ if the galaxies were distributed uniformly (background galaxies). The $N_0(r_0/r)^\gamma dv$ term
gives the number of correlated galaxies in the volume dv. Based on previous studies, the parameters were assumed to have values of $N_0 = 0.01$ Mpc$^{-3}$, $\gamma = 1.8$, and $r_0 = 5$ Mpc. Evaluation of the volume integral for an average depth of 1500 Mpc for the redshift galaxy and a circle radius of 1 Mpc yielded values of $126 \pm 15$ and $11.0 \pm 3.3$ for the number of background and correlated galaxies respectively. This corresponds to a signal-to-noise ratio of 0.94 for a single measurement (only one redshift galaxy was considered in the calculation). Since there are approximately 100 redshift galaxies in the field with redshifts close to 0.5, the signal-to-noise ratio would be 9.4 for the entire sample. The relatively high signal-to-noise ratio suggested that a significant number of correlated galaxies could be measured at a depth of 1500 Mpc.

The calculation of the number of background and correlated galaxies was extended to include magnitude dependence by integrating an assumed luminosity function. Calculating the number of galaxies in the circle as a function of apparent magnitude allows one to estimate how the galaxies will be distributed in the various magnitude bins. From these values, the faint end limit for the sample can be determined. The luminosity function parameters were given values of $\alpha = -1$ and $\phi^* = 0.01$ Mpc$^{-3}$. Luminosity functions for other samples and clusters have $\alpha$ and $\phi^*$ values similar to those assigned for this calculation. The value of $L^*$ was somewhat arbitrary and was set at 1 solar luminosity unit. The radius of the circle around the redshift galaxy was changed from 1 Mpc to 0.34 Mpc in order to reduce the amount by which the circles in the sample will overlap. The 0.34 Mpc radius corresponds to a measurement where there is on average one correlated galaxy with luminosity $L^*$ within the circle. The results of this calculation (for a single measurement) are given in the following table:

<table>
<thead>
<tr>
<th>Apparent Magnitude</th>
<th>N(background)</th>
<th>N(corrleated)</th>
<th>Signal-to-Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.63</td>
<td>0.14</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>44.63</td>
<td>0.56</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>45.63 (L*)</td>
<td>2.2</td>
<td>1.0</td>
<td>0.56</td>
</tr>
<tr>
<td>46.63</td>
<td>8.8</td>
<td>1.9</td>
<td>0.58</td>
</tr>
<tr>
<td>47.63</td>
<td>35</td>
<td>2.4</td>
<td>0.39</td>
</tr>
</tbody>
</table>

The luminosity function parameters were calculated from a best fit to this theoretical data to measure how well the parameters can be determined. Since the parameters were assigned values for this calculation, the fit to the data returned these values. However, this calculation was performed to determine the probable errors in the parameters. The results of the fit were:

$\alpha = -1.02 \pm 0.02$
$\phi^* = 0.0100 \pm 0.0004$ Mpc$^{-3}$
$L^* = 1.00 \pm 0.03$ solar luminosity units

**Conclusions / Future Considerations**

Given the signal-to-noise ratios (STNR) for the number of galaxies as a function of magnitude calculations, applying the correlated galaxy method to determine the
luminosity function of this deeper sample seems feasible. The low STNR for the high luminosity galaxies (L > L*) is expected due to their relatively low numbers. The STNR for the magnitude corresponding to L* (m*) and fainter magnitudes close to L* are acceptable. However, the STNR for galaxies with magnitudes that are fainter than m* by 2 or more are somewhat questionable. It may be possible to raise the STNR for these relatively faint galaxies by further adjusting the radius of the circle. The luminosity function parameters (α, L*, and φ*) are well determined when calculated from the theoretical data. This indicates that the sample should be adequate to yield an accurate luminosity function.

Since the method seems to be feasible, the next step is to use it to measure the luminosity function for this sample of galaxies. A computer program that will carry out the galaxy correlation calculations on the sample and produce number of excess galaxies as a function of magnitude data points is in the process of being written. A best fit to these data will give the luminosity function. Once the luminosity function is determined, it will be possible to investigate other properties of the galaxies in this sample.

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References
