Discovered over 100 years ago, RR Lyrae stars are now one of the best-known classes of variable stars. They fall within a specific area of the H-R diagram. RR Lyraes generally fall into two categories: longer-period stars pulsating in the fundamental mode, and shorter-period stars pulsating in the first overtone mode. A relatively small number of RR Lyraes pulsate in both modes. In the Draco dwarf spheroidal galaxy, there were originally thought to be ten of these double-mode stars (Nemec 1985), but our new, more-detailed analysis nearly triples that amount. With data obtained from the US Naval Observatory in Flagstaff and the Wyoming Infrared Observatory, I looked at the deconvolved light curves for all the first overtone stars to see if they exhibited evidence of fundamental mode pulsation as well. My supervisor for this project was Dr. Horace Smith.

I. HISTORICAL BACKGROUND

In the late 1890s, Solon Irving Bailey first recognized RR Lyrae stars as a separate class of variable stars. Their relatively short periods set them apart from the classical Cepheid variables. The RR Lyraes have a period averaging about half a day, whereas the Cepheids typically have week-long periods. RR Lyraes are now the most abundantly-known class of variable stars, accounting for over twenty percent of variable stars in the Milky Way, over seven times more than the Cepheids. RR Lyraes are not only found within our galaxy, but also in small sets of stars surrounding the Milky Way and in other galaxies as well. In globular clusters in our galaxy’s halo, over ninety percent of the variable stars are RR Lyraes. It is for this reason that RR Lyraes were originally called cluster-type variables. However more are now known in the galactic field than in these clusters. While about 7,000 RR Lyraes have been identified in all parts of the sky, it has been estimated that there are perhaps 80,000 left to be discovered.

In a Hertzsprung-Russel (H-R) diagram, which plots the absolute magnitude against the color or temperature for each star, we can see that...
the RR Lyraes fall within a well-defined area (see Figure 1). Along with the Cepheids and a couple other classes of variable stars, they are found in what is known as the instability strip. These stars become unstable and pulsate as they go through the core helium-burning stage. Also, if we make an H-R diagram of a typical globular cluster, we see that the RR Lyraes fall along what is called the horizontal branch (see Figure 2).

Observational data (ours is roughly 50 observations taken over a two-year interval) can be condensed into a one-cycle lightcurve based on a given period. In looking at lightcurves of many RR Lyraes, Bailey categorized the stars into three groups: of the longer-period stars, the high-amplitude a-types and low-amplitude b-types, and then the shorter-period c-types. Examples of ab-type and c-type lightcurves are shown in Figure 3. If we make a period-amplitude diagram of the RR Lyraes, we see that there is no sharp separation between the a- and b-types (see Figure 4). For this reason, we now combine these two groups into the long-period ab-type RR Lyraes. Over ninety percent of known RR Lyraes are ab-type.

Figure 1. A Hertzsprung-Russel diagram, showing the locations of some types of variable stars. Graphic from Smith (1995)

Figure 2. An H-R diagram of the Draco dwarf spheroidal galaxy. The approximate positions of RR Lyrae stars are indicated by the ellipse.
Around 1940, it was hypothesized and later confirmed that the RRab stars pulsate in the fundamental mode and the RRc stars pulsate in the first overtone mode. In the fundamental mode, the radius of the star increases and decreases periodically. In the first overtone mode, the average radius remains constant, but the surface is in constant motion. The radius of half of the star increases as the other half decreases. Each mode is equivalent to its one-dimensional simplification in a vibrating string.

It was observed in the early 1980s that the lightcurves of a few of the RRc stars exhibited more scatter than the others. Careful analysis revealed that these stars were actually pulsating in both modes at the same time. These double-mode stars were given their own group, the d-type RR Lyraes.

The pulsation period for each mode is dependent upon the radius of the helium second-ionization zone. There is a specific range of radii that will sustain the pulsations. This range of radii is governed by the temperature of the star. If the star is at the cool end of this range, the helium second-ionization zone is about 90% of the way towards the surface of the star. It pulsates in the fundamental mode as an RRab. If the star is at the warm end of this range, the helium second-ionization zone is further out in the star, and it pulsates in the first overtone mode as an RRc. There is a very small range of temperatures between the warm and cool ends in which the star pulsates in both modes simultaneously as an RRd.
In each case, as the star loses energy to the ionization, it contracts and appears dimmer. Due to its momentum, the star contracts below the equilibrium point, building up the interior pressure and temperature. When the pressure is too high, it rapidly expands and appears brighter. The energy that was built-up in the compression is now free to escape. Thus, the helium second-ionization zone acts as a sort of valve, absorbing energy when it is cool, and giving it off when it is hot. This mechanism was named Eddington’s valve, after Arthur Stanley Eddington, who developed the mathematical foundation of radial pulsation theory around 1917. The ionization zones of other elements such as hydrogen and carbon have some effect on the pulsations, but not to as great a degree as the helium second-ionization zone.

The Draco dwarf spheroidal galaxy, also known as UGC 10822, is visible at right ascension 17:20:12.39 and declination +57°54’55.3”, which is between the head and body of the dragon (see Figures 5 and 6). It was discovered in the 1950s. The galaxy is of approximately 10th magnitude, so it is only visible with a good telescope. Spatially, it is roughly 80,000 parsecs, or about 280,000 light-years, away from the Milky Way (see Figure 7). It is gravitationally bound to our galaxy, and therefore orbits it with a period of several billions of years.
II. ANALYSIS

To begin, I used a Fourier analysis program (affectionately referred to by the people working around me as “that beeping program”) to remove specified periods from the observational data. For each star, I ran the data through twice — once removing the fundamental period and once removing the overtone period. Then in the Image Reduction and Analysis Facility (IRAF), I used the Phase Dispersion Minimization (PDM) tool to analyze the lightcurves of both of the residual data sets. This data analysis technique was developed by Stelllingwerf (1978).

PDM plots a statistical value $\theta$ against different periods. Theta is a measure of how much the data scatters about the mean curve of a given period. As I looked at each residual data set, I looked to see if there was a low theta-value at the correct period. With the data that had the fundamental period removed, I checked to make sure that the lowest theta value was indeed at the specified overtone period. With the data that had the overtone period removed, I looked to see if the lowest theta value was at the fundamental period that had already been found. If no period had yet been found, I simply looked to see if there was a low theta value in the correct period range.

Of all the stars pulsating primarily in the first overtone mode, 10 were already-established RRd stars, 16 had fundamental periods newly found by Dr. Smith, 3 had been analyzed without finding fundamental periods, and 28 had not been analyzed. I found that one of the original 10 had a second possible value for the fundamental period. Of the 16 newfound double-modes, I concluded that the values already found were valid. In the 3 that had been analyzed without success, I found one that showed a possibility of a fundamental period, but there was so much scatter that I didn’t include it in my final results. Of the 28 unanalyzed stars, there were 4 which had low theta-values in the correct period range; however, none of these had clear enough lightcurves to be included in the final results. The lightcurves for one of the newfound RRd stars are shown in Figure 8. The period values are listed in Table 1.

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Figure 8. The lightcurves for V6117 are shown.
I looked at the lightcurves for each star which showed a fundamental period and estimated the amplitudes. I calculated the amplitude ratios for each star and compared the values of the original 10 with those of Nemec (1985). I found that they were all reasonably close except the one for which I found a second possible fundamental period. The only problem I had with estimating the amplitudes was that most of the curves were very noisy and therefore yielded high errorbars. Nonetheless, when I plotted the amplitude against the period, the data points were generally in the correct areas. The values I calculated for the amplitude ratios are listed in Table 1.

### III. Final Remarks

In the study of many sets of stars in and around the Milky Way containing RR Lyraes, it has been realized that they fall into two groups, depending on their metallicity. The two groups are called Oosterhoff groups, named after Pieter Theodorus Oosterhoff, who drew attention to this dichotomy in 1939. The Oosterhoff I sets tend to be more metal-rich (or rather less metal-poor) than the Oosterhoff II sets. This seems to have some correlation with the average periods and also with the percentage of RRc stars. In Oosterhoff I sets, the fundamental period averages around 0.55 days, the first-overtone period averages around 0.32 days, and RRc stars account for about 17% of all RR Lyraes. In Oosterhoff II sets, the fundamental period averages around 0.64 days, the first-overtone period averages around 0.37 days, and RRc stars account for about 44% of RR Lyraes.

In the Draco dwarf spheroidal galaxy, it appears that most of the RRd stars fall within the Oosterhoff II ranges (See Figure 9). However, there is one RRd that seems to belong to Oosterhoff I. This is not a newfound double-mode: it is one that Nemec identified in 1985.

![Figure 9. A Petersen diagram showing the periods of RRd stars in the Draco dwarf spheroidal galaxy in relation to those of Oosterhoff II clusters M15 and M68 and Oosterhoff I cluster IC4499.](image-url)
certain sets of stars within our galaxy and outside it as well. They also increase our understanding of stellar evolution and pulsation theories. Furthermore, double-mode RR Lyraes are perhaps even more important because their physical properties can be determined independently of stellar evolution theory. We can use stellar pulsation theory to determine their masses, and then compare the results of the two theories. For instance, from a Petersen diagram of RRd stars (as in Figure 9), we can determine their masses. Oosterhoff I RRd stars have a mass of about 0.65 times that of the sun, and Oosterhoff II RRd stars have a mass of about 0.75 times that of the sun. This diagram is named after Jørgen Otzen Petersen, who pointed out this relationship in 1973.

There are still some aspects of double-mode pulsation that astronomers do not quite understand. For instance, RRd stars are found more often in Oosterhoff II clusters, but there are still some Oosterhoff I clusters that have many and some Oosterhoff II clusters that apparently have none. Also, no one knows exactly what causes the double-mode pulsation. Nevertheless, we must continue to study RR Lyrae stars, as they give us insight into the history of the Milky Way and the universe.

REFERENCES
Nemec, James M. 1985, AJ 90, 204.