Gamma Decays in Neutron-Rich Nuclei

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Abstract

The nuclear shell model has been applied to calculate possible gamma decay schemes for neutron-rich nuclei, as well as various information about the individual energy levels within the nucleus. Comparisons will be made to experimental data if it exists. Information calculated about individual decays includes initial and final excitation energies, angular momenta, branching ratio, and mean lifetime.

Contents

Introduction

On any well-formed chart of the nuclides, one will find about 270 isotopes that are stable. These stable nuclei make up less than 0.5% of all known nuclei. Of the other 99.5% of nuclei, many are so exotic or hard to produce that few or none of their properties have been observed in any appreciable detail (See fig. 1). Due to current limitations in both theory and technology, it is most reasonable to study the properties of light to medium-light nuclei near stability, and extend the applications of theory out to the proton and neutron drip lines, where nuclei become unstable to proton and neutron decay, respectively.

There are many viable candidates for an effective theoretical solution to the nuclear many-body system. Two such models are the liquid drop model and the nuclear shell model. The liquid drop model treats the nucleus as if it were a charged liquid droplet held together by surface tension. In the nuclear shell model, nucleons are treated as interacting with a "mean field" caused by the other nucleons in the nucleus. This model is quite analogous with the model of the atom, as nucleons are thought to exist in discrete energy levels within this field. The greatest success of the nuclear shell model is that it explains the strong binding of energies with "magic numbers" of nucleons by stating that those nuclides represent completely closed shells. From here on, I will consider the nucleus according to the shell model.

Throughout the rest of this text, I will discuss various important applications of the shell model. More specifically, I will discuss how the shell model can be analyzed to determine the gamma decay scheme of excited nuclei near the neutron drip line. For the time being, my analysis will be limited to the rather small group of nuclides from Z=6 to Z=12, and from N=12 to N=20. Inferences will be made from these nuclides and applied to more massive nuclei in the future.

The Model Space and Hamiltonians

The model space is a computational concept that describes the orbitals and any truncations made within that set of orbitals which are assumed for a given calculation. As the number of nucleons within the nucleus increases, so to does the number of orbitals to be taken into account for any calculations, and similarly the amount of time and processing power necessary to conduct such calculations. Although it would be ideal if the full space could be taken for any calculations, it is highly impractical and, in some cases, impossible. The truncation that is most relevant to the data presented in this text is called the 0hω configuration, and it consists of the partition for which the least number of nucleons are in the valence shell with accordance to the Pauli principle. The above notations are important to remember when viewing the gamma decay diagrams that accompany this text and those available online.

Based on the model of nucleons as interacting with a mean-field created by the rest of the nucleus, explicit calculations of energy orbital properties can be made using the Hamiltonian operator. For the region of nuclei studied here, the non-relativistic Skyrme Hartree-Fock approximation and the relativistic Hartree approximation to the Hamiltonian are most useful. Although it would be unwise for me to go into great detail on these approximations, some qualitative view of the methods used to obtain my results should and will be given.

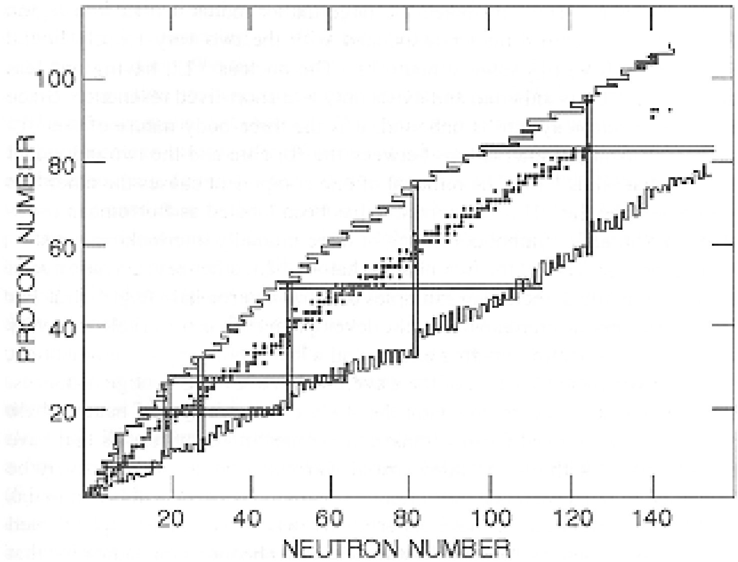

Figure 1: Chart of nuclides. Stable isotopes are shown as black dots. Nuclei extend above the line of stability to the proton drip line, and below to the neutron drip line. Magic numbers of protons and neutrons are show as the horizontal and vertical lines. For this review, light nuclei below A=40 are considered. (Courtesy American Institute of Physics)

Lifetimes and Linewidths

Once the Hamiltonian matrix has been applied to a model space, various information about the nucleus is determined. This information includes the absolute energies of potential levels, their parities, and their respective angular momentum value, as well as the isospin of the nucleus. It is now simply a matter of sorting to determine in what order the levels go, and what their respective excitation energies are. To take things one step more, it is now possible to determine which transitions between energy levels are possible. Though this information is useful, it is hardly enough on which to base any important conjectures about the nuclide in question, and we still know nothing about the actual gamma decay scheme of the nuclide.

When an exotic nucleus is created, it is often created in an excited state. If the energy of this excited state is sufficiently high, the nucleus becomes unstable to neutron decay and readily decays into a lighter nuclide. However, below the separation energy, denoted S_n, the nucleus undergoes a series of gamma emissions until it reaches the ground state. As the nucleus is still inherently unstable, it will in time beta decay from the ground state into a different nuclide, and the process repeats until a stable nuclide is formed. A general schematic of this is shown in Fig. 2. It is helpful to note that the gamma decay of a nucleus is analogous to the electromagnetic radiation of an electron in an excited-state atom. The main difference is that, when dealing with nuclei, much higher energies are involved, and therefore much larger frequencies.

There is much more to a gamma decay than simply the initial and final energy level. Several other pieces of information can be extracted once the energies, angular momenta, and matrix elements are known. One can also determine the mean lifetime of a particular state, and

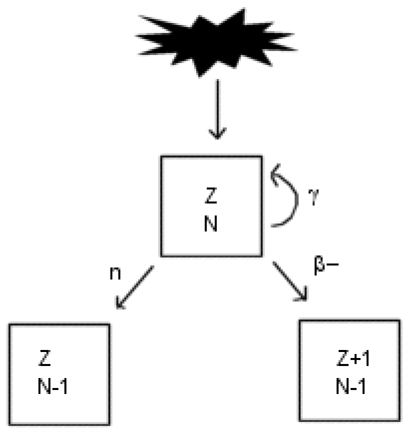

Figure 2: Possible decay routes for an unstable nucleus. If the energy is above the separation energy, neutron decay occurs. If the energy is below the separation energy, but above the ground state, gamma decay occurs until the nucleus is in the ground state. Eventually, the unstable ground state nucleus beta decays, essentially converting a neutron to a proton.

likewise the branching ratio of a given transition. Gamma transitions come in several different types, depending on the parities of the initial and final states, and the L-value of the decay. For even-parity decays, the L=1 transitions are denoted M1 (magnetic dipole) and the L=2 transitions are denoted E2 (electric quadrapole). For odd-parity transitions, the L=1 transitions are denoted E1 (electric dipole) and the L=2 transitions are denoted M2 (magnetic quadrapole). M2 transitions are almost always very weak, and therefore do not need to be included for a discussion of the gamma decay scheme. For now, we shall only consider even-parity decays. For M1 decays in such nuclei, the following relation gives the partial lifetime, in picoseconds:

$$\tau_a = \frac{0.0568 \cdot (2J_i + 1)}{E_\gamma^3 \, | M_{ab} |^2}$$

where J is the initial angular momentum, E is the energy of the gamma ray, and M is the matrix element for the transition. For E2 decays, the partial lifetime is given by the following:

$$\tau_a = \frac{816 \cdot (2J_i + 1)}{E_\gamma^5 \, | M_{ab} |^2}$$

Once the partial lifetime of all decays from a given level has been calculated, it is very simple to calculate the mean lifetime of the state, the half-life, and the branching ratio. The mean lifetime is related to the partial lifetimes by the simple relation

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$

and the half-life is related as $T_{1/2}=\tau \ln(2)$. The branching ratio is defined as $\frac{\tau}{\tau_a}$. Clearly, this relation shows that, for a given state, those decays that occur quickest (have the shortest lifetime) are also the most intense (have the largest branching ratio). Also, one must note that if a decay occurs by both M1 and E2 channels, the total branching ratio is the sum of the individual ratios.

Building a Database

Once the theoretical and computational groundwork has been laid, it is merely an exercise in computer programming to build a library of decay schemes. At this point, all the essential physics behind the schemes have been presented and, hopefully, understood. Once the shell model has been applied to a theoretical collection of neutrons and protons, and the possible states of this model have been computed, a file is created which contains thirteen rows of information, in this order: E_a, M_{ab}, J_a, T_a, J_b, T_b, E_b, L, N_a, N_b, P_a, P_b, and ECOUL. Both the T columns and the N columns (isospin and state number) can be ignored, as well as ECOUL. As all of the nuclei dealt with in this report are of positive parity, both P columns can be ignored as well. M_{ab} is the matrix element that will be used to calculate the lifetimes and branching ratios. The *a* and *b* states denote either the initial or final states of a gamma decay. Since we are dealing with decays, the final state must be lower in energy than the initial state, and this determines whether the *a* or *b* state is the final state. As this file can be thousands of lines in length, the only reasonable way to handle it is to write a computer program.

This program must accomplish the following things: (1) it must determine which state is the initial and which is the final, (2) it must organize the decays from the ground state up, (3) it must use the information to calculate the lifetimes and branching ratios, and (4) all of these must be output to a text file, for future manipulation. I have written a program in Fortran 90 that accomplishes all of these tasks. It first reads through the data file line-by-line, and eliminates those entries in which the *a* state and the *b* state are the same. For those entries in which this is not the case, it determines which state has a lower energy and enters all necessary values into an array in the appropriate order. Once all of the meaningful data has been read, it executes a simple two-layer sorting algorithm to sort the entries by E_i, and then by E_f. Finally, before any calculations are made, any duplicate entries are removed. It is also helpful, as we shall see later, to convert the energies from absolute energies to excitation energies relative to the ground state. Now that the main array is sufficiently formatted, calculations can be performed using the above formulae. The first and most basic calculation is to determine the partial lifetime for each transition. Once all of the partial lifetimes are determined, the mean lifetime per level can be calculated. And once the mean lifetime of a level has been determined, the branching ratio of each transition can be calculated in turn.

The final step of this calculation program is to output all relevant information to a file. There are many different formats that may be used to accomplish this end, and all are equally valid. I chose to display the ground state absolute energy as the first line, and then to group the decays by their initial energy level. Data is presented in the following order: E_{xi}, E_{xf}, J_i, J_f, B (a matrix element used for debugging purposes), τ_a, and BR_a. After the data has been displayed for a particular transition, be it M1, E2, or both, a line is output giving the total branching ratio BR for that transition. Once this has been done for all transitions from a given level, a line is output giving the mean lifetime of that state, followed by a blank line. This process is repeated for all entries in the data file. At this point, it is possible to determine the gamma decay scheme by reading the output file, though it would be a very tedious and awkward process. To accomplish this end, it is beneficial to use another program to convert a text-based output to a graphical interface.

Visual Representation

To keep analogous to the atomic shell model, a similarly formatted output would be quite beneficial. Energy levels would be displayed as horizontal lines at the appropriate scaling, and decays would be displayed as vertical arrows from the initial to the final state. Information such as the excitation energy, angular momentum, and mean lifetime would be most useful if displayed as text labels to the energy levels. As the branching ratio is parallel to the intensity of a decay, it is most useful to give a rough estimate of the ratio as the thickness of the arrow representing the decay. For such a graphical display, a language such as Adobe PostScript would be ideal; however, PostScript has no means of interactive input. Solving this issue requires a bit of programming that seems intuitively redundant: one can write a program in one language that writes the code for another language. As I had already worked with Fortran 90 for this project, I chose to write a program that inputs the data output from the previous program and outputs the code for a PostScript file. This is a bit clumsy but, as all that must be displayed is straight lines and text, it is easy to accomplish.

The details of the code written for this program are unimportant, so they shall be omitted in favor of a discussion of the interface and features allowed. The interface is text-based and quite simple, consisting of three input prompts. First, the names of the input and output file names must be specified, and then the user enters the energy at which he would like the diagram to truncate, usually S_n. The code is then executed and a PostScript file is created. There are many programs readily available to convert files from .ps to other formats, such as .jpg and .pdf, and that decision is left to the user. Features of my utility at this time include the option to set the minimum and maximum spacing for both the energy levels, in the vertical direction, and the decay lines, in the horizontal direction. The width of the decay lines is determined directly by the branching ratio, and is in 5 bins of a 20% step each. It is possible for the user to specify the maximum linewidth, and all other widths are calculated accordingly. Furthermore, to save space, decays that occur with a branching ratio of less than a certain percent (default of 1%) are omitted. These decays are usually so unlikely that they give no reasonable information about the true decay scheme. At the top of the file, information is displayed at the user's preference. For my sample files, the information at the top contains details of the model space and Hamiltonian operators, as well as values of the separation energy. Figs. 3, 4, and 5 contain several examples of files created using my program. The entire collection of files can be found in an online database at http://www.pa.msu.edu/people/santonocito/gamma.htm.

Future Work

Currently, data is only available in full for 7 nuclides. The database will continue to be built throughout the fall. Once the database is sufficiently completed, there are several paths that may be taken. Options may be added to display only Yrast states (those of increasing J-value). An upper limit may be placed on mean lifetime, as levels of high lifetime are near impossible to observe in the laboratory. Although it would require a more complex program, it would be possible to determine, given an initial excited state, all possible paths to ground and their likelihood. This concept could be extended to calculate the most probably life cycle of a created nucleus, incorporating neutron and beta decays, to stability.

Figure 3: Sample gamma decay schemes, for Carbon 17-20. These have relatively low S_n values, and therefore there are few bound states. For Carbon 19 (bottom left), the listed value of S_n is 160 keV, which would allow for no excited states; however, recent observations have allegedly shown the existence of the decay from the second to first excited states. For this reason, the first two states have been included despite having energies above the listed value of S_n.

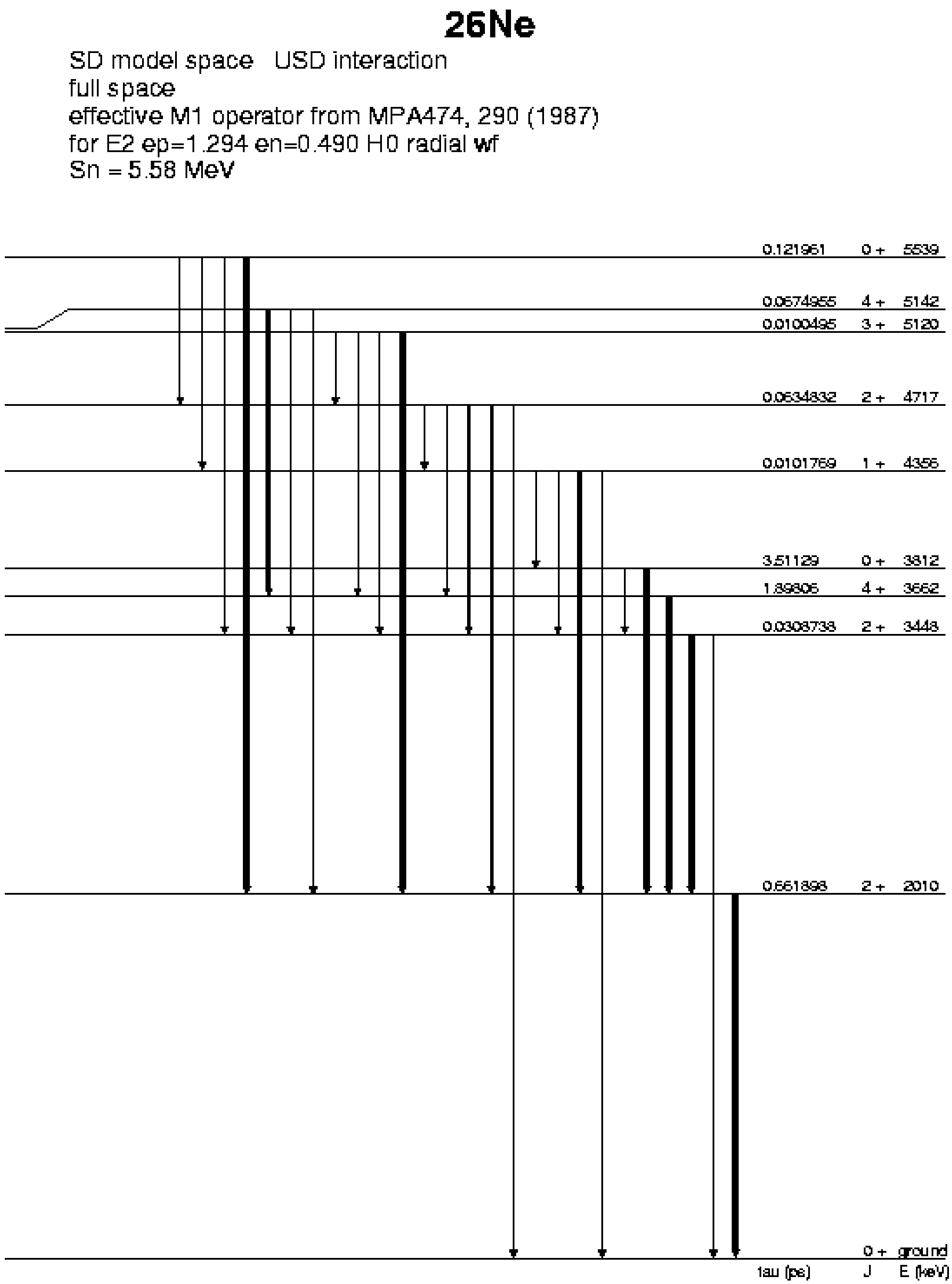

Figure 4: Decay Scheme for Neon-26.

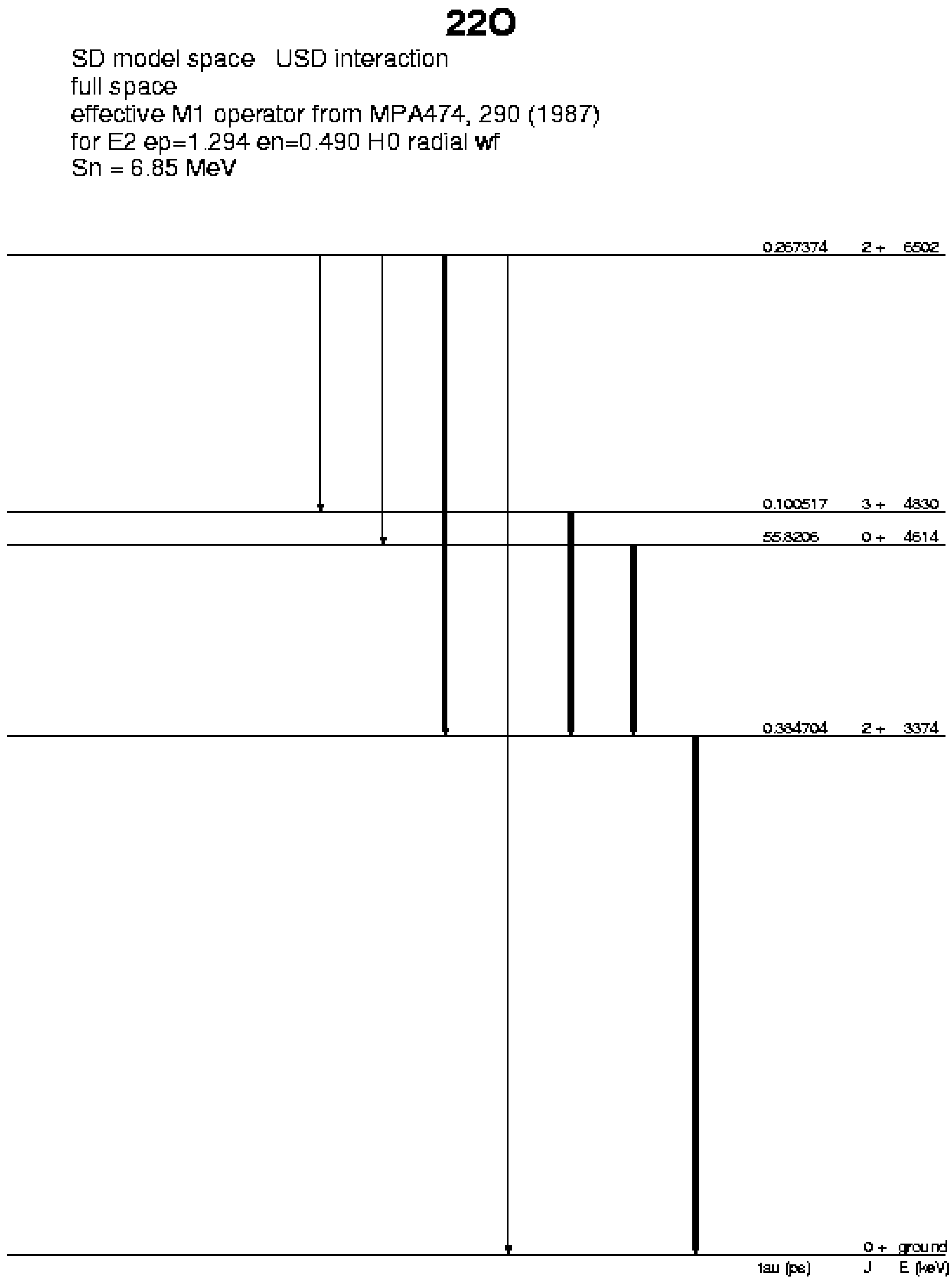

Figure 5: Decay scheme for Oxygen-22. The decays from the J=3+ to the J=2+ and from the J=2+ to the ground have been experimentally confirmed.

Comparison to Experiment / Conclusion

The benchmark of any model is its agreement (or lack thereof) with experiment. Approximately one month ago, data was taken for Magnesium-32 nuclei at the NSCL. Fig. 6, below, is a rough display of the resulting spectrum, compared with Fig. 7, my determination for low-lying states in Mg-32.

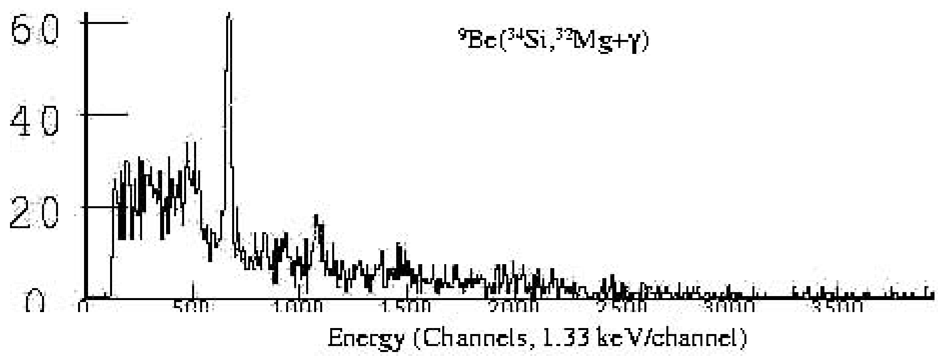

Figure 6: Magnesium-32 gamma decay spectrum. Note the sharp spike at approximately channel 700, or E=930 keV.

The sharp spike is most definitely the transition from the 2+ state to the ground state. A closer inspection of Fig. 7 reveals that there are many high intensity decays into the 2+ state, and this 2+ state is the only state which has a high intensity decay to ground. In other words, most paths to ground must pass through the 2+ state, causing a very large number of transitions from 2+ to ground. Similarly, smaller peaks at about 1200 keV and 2000 keV correlate to high-intensity transitions on the diagram below.

As stated previously, this project is still a work in progress. As more information becomes available, it will be added to the online database, and perhaps as more experimental information becomes available, new databases may be added with additional information. Comparisons to known nuclei have shown promise, and hopefully predictions made using this model will be equally as pleasing when compared to future experimental results.

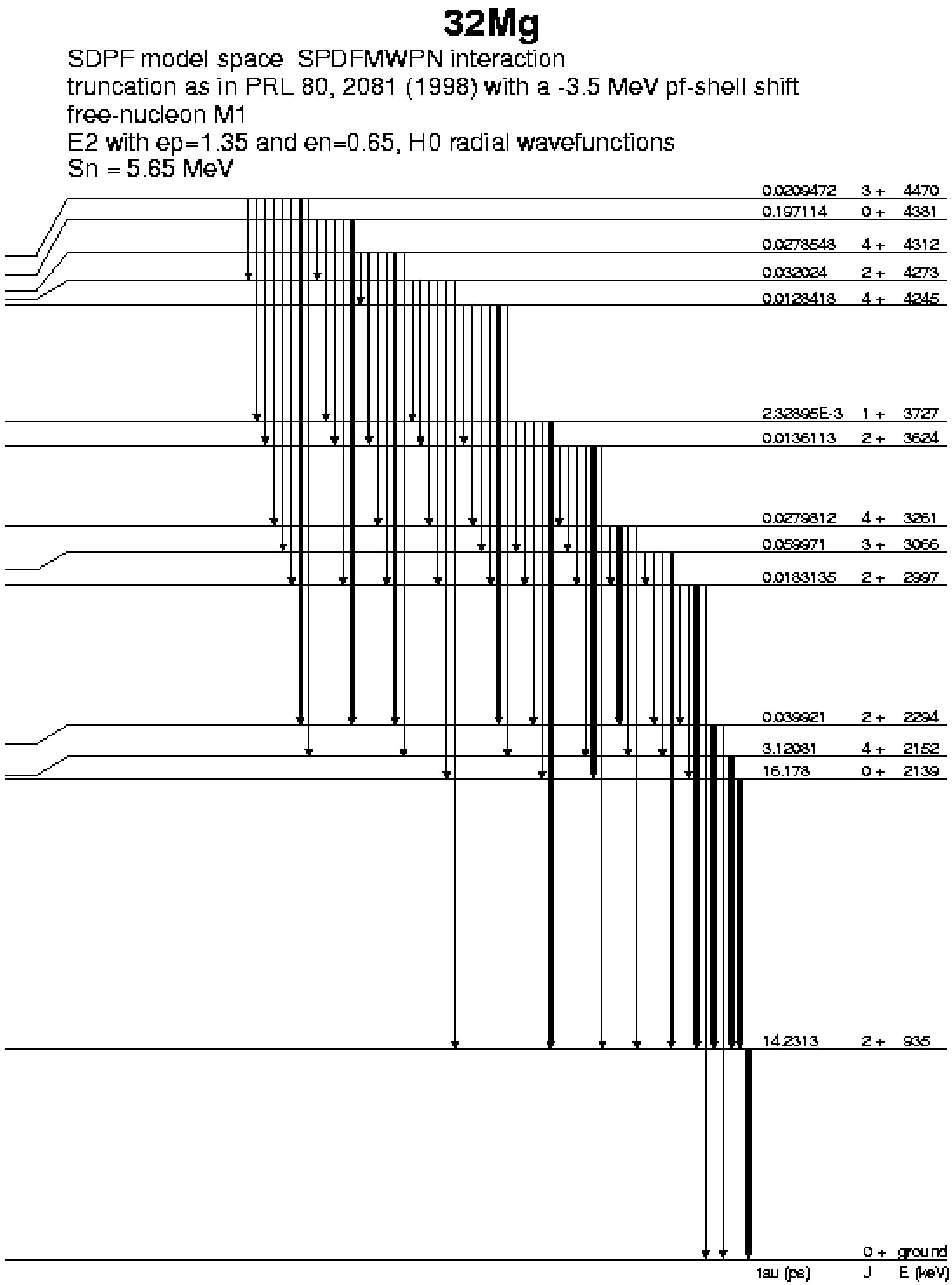

Figure 7: Magnesium-32. The energy of the first excited state (2+) is 935 keV.

References

Audi, G. & Wapstra, A. H., Nucl. Phys. A595, 409 (1995).

Brown, B. A., Prog. Part. Nucl. Phys. 47, 517 (2001).

Heyde, K., *From Nucleons to the Atomic Nucleus*. Springer-Verlag 1198.

Sorlin, O., Nucl. Phys. A685, 186 (2001).