USING BALANCE FUNCTIONS TO LOOK FOR QUARK-GLUON PLASMA

By

Michael Skoby

Supervisor:  Scott Pratt

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Michigan State University
Abstract

A new theoretical phase of nuclear matter is predicted from the laws of QCD, the quark-gluon plasma (QGP), where quarks and gluons are liberated from their usual hadronic confinement. It is expected that this is produced in heavy-ion collisions, where hadrons might not form until many fm/c after the collision takes place. Balance functions are used to test this theory by correlating charge-anticharge pairs with each other and determining when hadronization actually takes place.

What is QGP?

Conventional phases of matter are solids, liquids, and gases. These states of matter experience phase transitions when they change from one phase of matter to another with different properties over an insignificant change in temperature or pressure. For example, as energy is added to a solid its temperature rises. When it gets to a certain temperature it melts into a liquid and remains at the same temperature as energy is added until the solid becomes completely liquid. As energy is added to the liquid its temperature will rise until it reaches a boiling point. Then, the liquid will begin a phase transition as it starts to evaporate into a gas. The temperature of the liquid and gas will remain constant with constant energy being added until all the liquid becomes gas, thus completing another phase transition. QGP is a new theoretical phase of nuclear matter that exists at extremely high temperatures and density. It can be described as a collection of unbound quarks and gluons. It is believed that the universe consisted of QGP before the first 10 microseconds after the Big Bang [1]. Usually we observe nuclear matter as protons and neutrons, but QGP is a phase in which the components are the quarks and
gluons that constitute protons and neutrons. Furthermore, the condensed quark-antiquark fields in the vacuum are also melted at the QGP phase transition. Protons and neutrons couple to this field and receive most of their mass.

**How do we observe this new phase?**

Assuming that QCD is correct in that it suggests that ordinary matter can be transformed into QGP [1], we would be able to observe QGP in relativistic heavy-ion collisions. These are collisions that occur between two nuclei of heavy ions, such as lead or gold, at very high energies. After two nuclei smash together head-on, the protons and neutrons break apart from the impact of the collision and create a dense blob of particles. This collection of particles provides the high density and temperature necessary for the QGP transition to take place. The phase transition of normal matter to QGP transpires similarly like water to steam. There are many experimental signs from these heavy ion collisions of the QGP, such as strange-quark enhancement [2] and suppression of certain particles in gold-gold collisions [3].

**What is a balance function?**

In a heavy-ion collision thousands of charge-anticharge pairs are created during the collision, which lasts 10-20 fm/c. Since charge conservation is local, every positive charge is accompanied by a negative charge. The pairs that are formed shortly after the collision are separated much further in relative momentum due to the high temperature and large velocity gradients created after such a collision. The purpose of a balance function is to quantify this separation correlation between these charge-anticharge pairs. The balance function is binned according to the relative momentum of a balancing particle-antiparticle pair. The balance function is defined as,
\[ B(p_2|p_1) = \frac{1}{2} \{ \rho(+,p_2|-,p_1) - \rho(+,p_2|+,p_1) - \rho(-,p_2|+,p_1) - \rho(-,p_2|-,p_1) \}, \]

where \( \rho(+,p_2|-,p_1) \) is the conditional probability that a positively charged particle will be in a bin \( p_2 \) while a negatively charged particle will be in a bin \( p_1 \) in the same event [4]. The first term, \( \rho(+,p_2|-,p_1) \), accounts for all pairs of positive and negative particles measured in one event. This includes the pairs that are not related at birth by charge conservation. By subtracting \( \rho(+,p_2|+,p_1) \) which is constructed using all pairs of particles with the same sign, the uncorrelated pairs are statistically subtracted from the distribution.

In principle, \( p_1 \) and \( p_2 \) could label any momenta for the two particles. For example, \( \rho(\pi^+,p_2|\pi^-,p_1) \) is the probability that a positively charged pion exists in bin \( p_2 \) while at the same time a negative pion exists in bin \( p_1 \). To determine these probabilities we count the number \( N(b,p_2|a,p_1) \) of pairs that satisfy both conditions and divide by the number \( N(a,p_1) \) of particles that are of type \( a \) and in bin \( p_1 \). In our studies \( p_1 \) will correspond to the first particle being anywhere in the detector and \( p_2 \) will refer to \( Q_{\text{inv}} \), the difference in the momentum between the pairs.

**Why do we want to make balance functions?**

We can utilize the balance function to help find clues to whether or not QGP is formed in a heavy-ion collision. Electric charge, strangeness and baryon number are carried by quarks. The balance function can be used to determine whether quarks were produced early, \( \tau < 1 \text{ fm/c} \), or in a late-stage hadronization [4]. The width of the balance function can be used to determine separation between charge-anticharge pairs. Since most of the charge is created at hadronization, we can determine the time scale at which hadronization takes place. If hadronization happened early we would expect a broad
width in the balance function since pairs created in early stages have more time to separate before the reaction ends. Early hadronization implies there was no QGP. However, a narrow width in the balance function would indicate that hadronization happened late, potentially allowing the QGP to develop. Therefore, the motivation in finding these balance equations is to find valuable insight to the possibility of QGP formation.

**Calculating Partition Functions**

In order to determine the balance functions we need to simulate the thermal production processes in heavy-ion collisions. We do this by randomly producing particles through means of Monte Carlo. The particles produced in each event are dependent on the weight (likelihood) of each particle. The equation for these weights is given by the recursive relation,

\[
W_j = \frac{1}{A} \frac{a_j \omega_j Z_{A-a_j,Q-q_j,B-b_j,S-s_j}}{Z_{A,Q,B,S}},
\]

where \( a_j \) is the number of decay products of the jth particle and \( Z_{A,Q,B,S} \) is the canonical partition function. The canonical partition function is the summation over the Boltzmann factor \( e^{-E_{\alpha}/T} \) for all states \( \alpha \) for which the total number of particles in the system, \( A \) is constant.

\[
Z = \sum_{\alpha} \langle \alpha, A, Q, B, S | e^{-E_{\alpha}/T} | \alpha, A, Q, B, S \rangle,
\]

where \( Q \) is the net electric charge of the system, \( B \) is the net baryon number of the system, \( S \) is the net strangeness of the system, \( E_{\alpha} \) is the energy of the \( \alpha \)th state and \( T \) is the temperature. This function can be represented as a recursive relation given to be,
\[ Z_{A,Q,B,S} = \frac{1}{A} \sum_{\text{all species}} \omega_j Z_{A-q_j,Q-B-b_j,S-s_j}, \quad (3) \]

where \( q_j \) is the electric charge of the jth species, \( b_j \) is the baryon number of the jth species and \( s_j \) is the strangeness of the jth species. The term \( \omega_j \) is defined as,

\[ \omega_j = \frac{(2s + 1)}{\hbar^3} V \left( \frac{mT}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{m}{T}}, \quad (4) \]

where \( s \) and \( m \) are the spin and mass respectively of the jth species, \( V \) is the volume and \( T \) is the temperature. Although this form for \( \omega_j \) is non-relativistic, we incorporated a relativistic form that contains Bessel functions. Before we can use the recursive weight equation we must first calculate the recursive partition function for all systems with particles fewer than the system we want to observe. The partition function returns the number of energy states given a specified number of particles and total charges in a system. We know that a system without any particles must have a net charge of zero for all charges and the system has one energy state so we have the initial condition that \( Z(0,0,0,0) = 1 \). We increase the total number of particles by one and calculate \( Z(1,\ldots) \) using the iterative equation above. We then use these values to calculate the next iteration of the partition function, but we only concern ourselves with systems in which the absolute values of the net charges are less than or equal to the total number of particles in the system.

**Producing Particles**

We want to randomly select particles (Monte Carlo) from a system that has A number of particles and zero net charge for all charges. Determining what particle is chosen is dependent on its weight as explained earlier. We calculate the weight of each
possible particle and add them together until the total weight is greater than a chosen random number. When the weight is greater than the random number the particle is considered observed and its momentum is calculated. When a particle is observed it is no longer considered in the system since we know what it is and what its momentum is. Therefore, the total number of particles and the net charges are decreased based on the number of decay products and charges of the selected particle. This modification will change the weights of each particle since the system is changed. The process above repeats until there are no particles left in the system and this completes one event. For the results to have any meaning there must be many events to get a wide distribution of particles.

**Hadronic Gas**

The system described above is a hadronic gas (HG) because the possible particles in the system are hadrons. They feel only the strong interaction governed by QCD. Hadrons are composite particles that are made up of quarks or quarks and antiquarks. They are separated into two classifications: mesons and baryons. Mesons are particles that consist of a quark and an antiquark and baryons are constructed from three quarks. The six known and apparently only quarks are the up (u), down (d), strange (s), charm (c), bottom (b) and top (t) quarks. The particles in the HG are only made of either the up, down or strange quarks or a combination of two or three of them. The properties of these quarks dictate the properties of the particles they form. The up quark has a $2e/3$ electric charge and the down and strange quarks have a $-e/3$ charge. All three quarks have a baryon number of $1/3$ and spin of $1/2$. Antiquarks have the same properties only the
charges and baryon numbers have the opposite sign of their accompanying quark. For example the π mesons are explained as

\[ \pi^+ = (u\bar{d}) , \quad \pi^- = (\bar{u}d) . \]

Consider the positive pion. The charge is the summation of the charges of the up and antidown quarks:

\[ q_{\pi^+} = (2/3 + 1/3)e = e . \]

The baryon number is the sum of the baryon numbers:

\[ B = 1/3 - 1/3 = 0 . \]

It can be seen from similar calculations that all mesons have a baryon number of zero and all baryons have a baryon number of 1 and –1 if it is an antibaryon. The spins of the particles are the sums of the spins of their composite quarks. All the particles in the HG have a spin of either 0, 1/2, 1 or 3/2. Below are samples of mesons and baryons used in the system and their quark content. Spin-1 mesons and spin-3/2 baryons were also included.

Meson Table (all spins = 0)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark content</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ )(139.6)</td>
<td>( u\bar{d} )</td>
</tr>
<tr>
<td>( \pi^0 )(135.0)</td>
<td>( uu ) and ( d\bar{d} )</td>
</tr>
<tr>
<td>( \pi )(139.6)</td>
<td>( d\bar{u} )</td>
</tr>
<tr>
<td>( K^+ )(493.6)</td>
<td>( u\bar{s} )</td>
</tr>
<tr>
<td>( K'^+ )(497.7)</td>
<td>( d\bar{s} )</td>
</tr>
<tr>
<td>( K^0 )(497.7)</td>
<td>( sd )</td>
</tr>
<tr>
<td>( K )(493.6)</td>
<td>( su )</td>
</tr>
<tr>
<td>( \eta^\prime )(548.8)</td>
<td>( uu, d\bar{d} ) and ( ss )</td>
</tr>
</tbody>
</table>
### Baryon Table (all spins = 1/2)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark content</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(938.3)</td>
<td>uud</td>
</tr>
<tr>
<td>n(939.6)</td>
<td>udd</td>
</tr>
<tr>
<td>Λ(1115.6)</td>
<td>uds</td>
</tr>
<tr>
<td>Σ⁺(1189.4)</td>
<td>uus</td>
</tr>
<tr>
<td>Σ⁰(1192.5)</td>
<td>uds</td>
</tr>
<tr>
<td>Σ⁻(1197.4)</td>
<td>dds</td>
</tr>
<tr>
<td>Ξ⁰(1314.9)</td>
<td>uss</td>
</tr>
<tr>
<td>Ξ⁻(1321.3)</td>
<td>dss</td>
</tr>
</tbody>
</table>

### The Results

The balance function, $B(Q_{inv})$, was calculated as a function of the relative momentum, $Q_{inv}$. The results from the balance function are based on the assumption that a perfect detector observes the particles (perfect acceptance). This means that all the particles that we wish to observe in the system are detected and all the extraneous particles from the collision are not accounted for. In actual experiment, some of the particles are not detected due to limited acceptance (e.g. the STAR experiment at RHIC). The results from the simulations are graphed on the following page. The “pions only” graph shows the balance function when only pion pairs are generated. The “+reson.s” graph shows the balance function when pions are generated and accounts for resonances that produce pions when they decay. The “perfect acceptance” graph shows the balance function with decays when a perfect detector is assumed. It is the same as the “+reson.s” graph. The “STAR” graph shows the balance function with decays when STAR acceptance is assumed.
The widths were calculated and here are the results:

<table>
<thead>
<tr>
<th>Graph</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pions only</td>
<td>265.2</td>
</tr>
<tr>
<td>+reson.s</td>
<td>264.3</td>
</tr>
<tr>
<td>STAR</td>
<td>247.0</td>
</tr>
</tbody>
</table>

**Conclusions**

STAR has observed balance functions that narrow as collisions become more central, which supports the QGP hypothesis (See K. Hinko’s REU write-up). In our calculations we saw that the widths for pions only and pions with decays differs by less than 1%. Therefore, we can conclude that decays do not have a significant effect on the width of the balance function. Thus, the narrowing of the balance functions seen in experiment cannot be attributed to resonance decay. We also see that the STAR acceptance has a small but non-negligible effect on the width of the balance function. It can be accounted for in the simulation. In any case, all of our widths are narrow and thus formation of QGP cannot be dismissed from these results.
Bibliography


