SYMmetry Energy and Its Impact on Nuclei in the Thomas-Fermi Approximation

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Abstract
The binding energy of a nucleus can be found by solving the Thomas-Fermi equations. Solving these numerically allows for much information to be obtained. Several things can be explored, among these, the relationship between symmetry energy and neutron skin, the effect of the mass number on the neutron skin, and how the proton and neutron densities otherwise change under extreme symmetry energies or mass number, A.

Introduction
The binding energy is the sum of several different components, symmetry energy being the focus of this research. The equations below are the Thomas-Fermi equations (3).

\[ 0 = \frac{\tilde{\varepsilon}_p}{\rho_0} \left( \rho / \rho_0 \right) - a_1 \nabla^2 \left( \frac{\rho}{\rho_0} \right) + \frac{\delta E_{\text{sym}}}{\delta \rho_p} + \Phi - \mu_p \]
\[ 0 = \frac{\tilde{\varepsilon}_n}{\rho_0} \left( \rho / \rho_0 \right) - a_1 \nabla^2 \left( \frac{\rho}{\rho_0} \right) + \frac{\delta E_{\text{sym}}}{\delta \rho_n} - \mu_n \]

On the right hand side appear variational derivatives of net energy with respect to proton density and neutron density, respectively. By setting those expressions to zero, while adjusting \( \mu_p \) and \( \mu_n \), one can minimize the energy of a nucleus at fixed N and Z. Below is an expression for the symmetry energy contribution to the net energy, where \( s(\rho) \) is the symmetry energy per nucleon, written next schematically as energy of combined capacitor with capacitances proportional to \( \rho/s(\rho) \):
Using the programming language FORTRAN 77, the Thomas-Fermi equations are solved numerically for given nuclei, supplied to the program in its initial conditions. Previously, this program only accommodated a very basic form of symmetry energy. However, by introducing a few subroutines, the program is now able to accommodate any function \( s(\rho) \). The program used in this research project was written in units of GeV and fm, so unless otherwise stated, those are the units on any data presented in this paper.

**Neutron Skin and Symmetry Energy**

208\(^{\text{Pb}}\) is the most neutron-rich stable isotope, and therefore is a good nucleus to study the effects of symmetry energy on neutron skin. The neutron skin is the difference between the neutron rms radius and the proton rms radius. Figure 1 plots the neutron skin versus \( \gamma \), which is a parameter that was systematically changed in an expression for \( s(\rho) \). This expression is:

\[
s(\rho) = s_0 \left( \frac{\rho}{\rho_o} \right)^\gamma
\]
By changing $\gamma$ in small increments, it is clear that the neutron skin increases in thickness as $\gamma$ increases. Since $\rho$ is less than $\rho_0$, this means that as the symmetry energy decreases, the neutron skin becomes thicker.

**Neutron Skin and Mass Number**

The thickness of the neutron skin obviously depends on the number of neutrons in the nucleus. As shown in figures 2 and 3, which plot neutron skin versus mass number, $A$, for isotopes of sodium and tin, the neutron skin increases in thickness as the mass number increases. Because the number of protons is constant, increasing the mass number is solely increasing the number of neutrons. As expected, the neutron skin increases in thickness as more neutrons are added to the nucleus.
The range for the sodium isotopes in this figure is for A between 19 and 33.

The range for the tin isotopes in figure 3 is for A between 100 and 132.

**Proton and Neutron Densities at Extreme Symmetry Energies**

It is desirable to see how the proton and neutron densities behave at extreme symmetry energies. The two extreme symmetry energies are the superhard and supersoft symmetry energies. The superhard symmetry energy is 

\[ s(\rho) = 0.014 \left( \frac{\rho}{\rho_0} \right)^2, \]

and the supersoft
symmetry energy is \( s(\rho) = 0.0385 \left( \frac{\rho}{\rho_0} \right) - 0.021 \left( \frac{\rho}{\rho_0} \right)^2 \) [5]. The proton and neutron densities are plotted versus radius for these symmetry energies in figures 4 through 7.

Figure 4 plots the proton and neutron densities of 208Pb versus the nucleus’ root mean square (rms) radius for superhard symmetry energy. The neutron density is greater than the proton density for a superhard symmetry energy.
Figure 5 plots the proton and neutron densities versus the root mean square (rms) radius for supersoft symmetry energy. The neutron and proton densities are approximately equal for a supersoft symmetry energy.

![Proton profiles for 208Pb](image)

Figure 6 compares the proton densities of the superhard and supersoft symmetry energies. The proton density is greater for the supersoft extreme than for the superhard extreme.

![Neutron profiles for 208Pb](image)

Figure 7 compares the neutron densities of the superhard and supersoft symmetry energies. The neutron density is greater for the superhard extreme than for the supersoft extreme.
Proton and Neutron Densities at Extreme A

It is also desirable to see how proton and neutron densities behave close to the proton and neutron driplines. Figures 8 and 9 display proton and neutron profiles for $^{19}$Na and $^{35}$Na, respectively, with symmetry energy $s(\rho) = s_0 \left( \frac{\rho}{\rho_0} \right)$, where $s_0 = s(\rho_0)$. These isotopes were chosen because $^{19}$Na is the isotope of sodium along the proton dripline and $^{35}$Na is the isotope of sodium along the neutron dripline.

Figure 8 shows the proton and neutron profiles of $^{19}$Na, which is the isotope of sodium that lies along the proton dripline.
Figure 9 shows the proton and neutron profiles of $^{35}\text{Na}$, which is the isotope of sodium that lies along the neutron dripline.

![Proton profiles](image)

**Fig. 10**

Figure 10 compares the proton profiles for $^{19}\text{Na}$ and $^{35}\text{Na}$.

![Neutron profiles](image)

**Fig. 11**

Figure 11 compares the neutron profiles for $^{19}\text{Na}$ and $^{35}\text{Na}$.

**Conclusions**

Several trends can be observed from the various figures in this paper. The neutron skin becomes thicker as the number of neutrons increases. The neutron skin becomes thicker as the symmetry energy decreases. $\rho_p$ is greater than $\rho_n$ for $Z$ greater than $A-Z$. $\rho_p$ is less
than $\rho_n$ for $Z$ less than $A-Z$. $\rho_p$ is greater with a supersoft symmetry energy than with a superhard symmetry energy. $\rho_n$ is greater with a superhard symmetry energy than with a supersoft symmetry energy. $\rho_p$ is very close in value to $\rho_n$ for supersoft symmetry energy. $\rho_p$ is less than $\rho_n$ for superhard symmetry energy. The proton density is greater for neutron-poor nuclei, e.g. 19Na, than for neutron-rich nuclei, e.g. 35Na. The neutron density is greater for neutron-rich nuclei than for neutron-poor nuclei.

Symmetry energy has an effect on many quantities associated with nuclear binding energy, among those are the neutron skin, and proton and neutron densities. By investigating the relationship between symmetry energy and these quantities, a greater understanding of nuclear structure is achieved.

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References