Using Fourier Coefficients to Determine Metallicity of RR Lyrae Stars

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Abstract:

Fourier decompositions were performed on the light curves of RR Lyrae stars observed using the MSU Observatory telescope and SDSS’ Apache Point telescope. The relationship between metallicity and Fourier coefficients was then examined in the g’ filter.

I. Background on RR Lyrae Stars

RR Lyrae stars are pulsating variable stars whose surface layers show periodic expansion and contraction, which lead to changes in their magnitude. An RR Lyrae star is intrinsically variable, so the variation in its light output is due to physical changes within the star itself—i.e., its pulsation—and not to external properties, such as in the case of an eclipsing binary.

On a Hertzsprung-Russell diagram, RR Lyrae stars are found on the horizontal branch. Stars on the horizontal branch are powered by helium fusion in their cores, and ones within the instability strip exhibit pulsation. On the horizontal branch, the blue and red edges of the instability strip are located near \((B-V)_0 = 0.18\) and 0.40 [Smith 1995]. RR Lyrae stars are found here in the instability strip.

RR Lyrae stars exhibit a change in V magnitude between 0.2 and 2, and their periods are less than a day. The fairly short periods of these variable stars allows complete light curves to be formed within a relatively short time, making them easy to study.

One important characteristic of RR Lyrae stars is their use as standard candles because of their well-established period-luminosity relationship. RR Lyrae stars, being located on the horizontal branch, have approximately the same absolute magnitude, and using their absolute and apparent magnitudes, astronomical distances can be determined.

An RR Lyrae star can be identified by its light curve shape. The astronomer Solon I. Bailey was the first to divide these variables into three subclasses, based primarily on the shape of their light curve: Bailey types a, b, and c (see Figure 1), though these are usually simplified to types ab and c.

Figure 1. Diagram of the three Bailey types of RR Lyrae stars, classified according to light curve shape. Smith, 1995.
RR Lyrae stars of type ‘a’ display a rapid increase of light, followed by a slightly-less-rapid decrease; type ‘b’ exhibit a moderately-rapid increase of light, followed by a slower decrease; and type ‘c’ show a sinusoidal light curve shape, with nearly equal rates of increase and decrease.

II. Data Collection

A. Sloan Digital Sky Survey

The Sloan Digital Sky Survey (SDSS), the largest photometric and spectroscopic survey to date, began in 1998. It uses the 2.5-meter Apache Point optical telescope in New Mexico and aims to map 25% of the sky. As of the latest data release in June 2007, it has obtained observations on around 287 million unique objects and spectra for 1.3 million objects. RR Lyrae stars can be readily detected within such large-scale surveys because of their short periods and distinctive periodic light curve shapes.

SDSS takes images of astronomical objects using five filters: u, g, r, i, and z. While RR Lyrae stars have been well-studied in the Johnson-Cousins UBVRI photometric system, the most widely-used photometric system for past surveys, there is currently little data on RR Lyrae stars in the SDSS $ugriz$ system. Because the $ugriz$ photometric system is likely to be the standard system for various future surveys, it is important to understand the properties of RR Lyrae stars in this system. Figure 2 shows the passbands of the various filters used in both systems.

B. MSU Observatory

A 1024x1024 Apogee Alta CCD was used to take images of various RR Lyrae stars over a seven-week period using the 24-inch telescope at the MSU Observatory. The images were taken in six different filters: V, B, I, $g'$, $r'$, and $i'$. 

![Figure 2. Graph showing the different passbands for the $ugriz$ and UBVRI photometric systems.](image-url)
III. Reduction of CCD Images

The images taken by the CCD at the MSU Observatory needed to be processed using IRAF, the Image Reduction and Analysis Facility. As its name would imply, IRAF is used in the reduction and analysis of astronomical images. It includes packages that provide tools for basic CCD reductions.

Calibration data was used to remove additive effects from the CCD images. These effects include dark current and multiplicative gain across the chip. Calibration diminishes unwanted signal from the images.

Some CCD pixels will read out a value even if they are not exposed to light, so in order to remove the background of the CCD from the image, zero-second exposures were used to determine the zero-point of the CCD output. A median of all the bias frames were combined to form a single zero (bias) frame, which was then subtracted from the other frames.

With some CCDs, a non-negligible amount of background noise is added to the images during long exposures, so a dark frame was used to remove this dark current. Dark exposures (long exposures with the shutter closed) were taken and median-combined to form a single dark, which was then scaled to the exposure time of the data frames. Because no light hits the camera, a dark frame measures only the energy from the CCD itself (the dark current).

Last, the data must be flat-fielded. A flat field compensates for an uneven response in the pixels’ response to light, and flats must be taken in each filter used. The data was divided by the corresponding flat in order to remove any pixel-to-gain variations.

In IRAF, these reductions were accomplished using the task \texttt{ccdproc} in the package \texttt{imred}.

![Figure 3. Examples of a zero (bias) frame, a dark frame, and a g' flat frame.](image)

IV. Spectroscopy

Using a star’s spectra, information about the intrinsic state of a star can be determined, such as its radial velocity, chemical abundances, and temperature. Spectroscopy is especially fundamental to determining the metallicity of a star, or its heavy element abundance (the abundance of elements heavier than helium, which are also referred to as “metals”). This is often written in [Fe/H] notation, where the iron-to-hydrogen
ratio in the photosphere of one star is related to that ratio in another star, usually the Sun:

$$[\text{Fe/H}] = \log(\text{iron/hydrogen})_{\text{star}} - \log(\text{iron/hydrogen})_{\text{Sun}}$$ [Smith 1995].

A disadvantage of spectroscopy, however, is that spectra are very difficult and time-consuming to get. Also, since RR Lyrae stars are variable, the phase of the star at the time that the spectra are taken is important because the spectral lines will actually change with the phase of the star (see Figure 4).

Figure 4. Graph showing the change in spectrum from maximum light (top) to minimum light (bottom). Smith, 1995.

V. Photometry

Photometry is a less-difficult alternative to spectroscopy. Since spectroscopy involves taking the light from a star and spreading it out over the different wavelengths of a spectrum, more photons are needed in order to get high readings. With photometry, however, all of the light from one star is considered collectively, making it far easier to perform.

Because this data was comprised of uncrowded star fields, aperture photometry could be used. The flux from the star was then determined by drawing apertures around the star.

Figure 5. Reduced image of the RR Lyrae star TV Boo, showing the apertures around the variable star. The length of one arc-minute is illustrated by the green arrow.

In aperture photometry, circular apertures are drawn around the variable, reference, and check stars. The innermost circle is
drawn just around the star itself. This circle should include most of the light from the star itself but very little of the background sky. In Figure 5, this circle is indicated in green.

The two light-blue outer circles include the background light but not light from any neighboring stars. The outer sky value is subtracted from the inner star value, which gives the magnitude of the star.

The value for the RR Lyrae star is then subtracted from that of the comparison (or reference) star, as long as the difference between the comparison star and the check star doesn’t change (i.e., the comparison star is not variable). This method of differential photometry corrects for variable seeing, such as that caused by clouds or changes in air mass.

The task phot in the IRAF package apphot was used to do the photometry on these images.

VI. RRL Lyrae Data from 24-Inch

Over the course of the summer, enough data was collected at the MSU Observatory that relatively-complete light curves could be constructed for four RR Lyrae stars in the g’ filter: AV Peg, SW Aqr, SW Dra, and X Ari (see Figures 6-7).

All of these stars were of type RRab, and the sharp rise in magnitude followed by a slower decrease can be seen.
Figure 6. Light curves for AV Peg and SW Aqr in g' filter, with a Fourier sine series fit to each curve.
Figure 7. Light curves for SW Dra and X Ari in g’ filter, with a Fourier sine series fit to each curve.
VII. Fourier Analysis

The shape of each light curve can be described by a discrete Fourier series. In this case, a sine series was used:

\[ \text{mag} = A_0 + \sum A_i \sin[i\omega t + \phi_i]. \quad (1) \]

The series was of order 15 for AV Peg and SW Aqr, which had very complete light curves (see Figure 6). This is mainly due to their short periods, which made it easier to obtain data for these stars’ light curves. In the case of SW Dra and X Ari (see Figure 7), there were some gaps in the light curve data, which led to errors in the Fourier fit at higher orders. For these two stars, a series of order 8 was used. Because higher-order terms are smaller, it does not much affect the results of the Fourier fit to use 8th- or 15th-order series for different light curves.

Work by Jurcsik and Kovács (1996) shows that there is an empirical relationship between the metallicity of a star and the Fourier coefficients from its Fourier fit, at least in the V filter. In particular, the period of the star along with the \( \phi_{31} \) value, which is calculated by the equation

\[ \phi_{31} = \phi_3 - 3\phi_1, \quad (2) \]

were found to have a relationship with the metallicity:

\[ [\text{Fe/H}] = -5.038 - 5.394P + 1.345\phi_{31}. \quad (3) \]

Again, because the \( \phi_{31} \) value depends only on the values of \( \phi_1 \) and \( \phi_3 \), the difference in using a sine series to order 8 or to order 15 is negligible.

The period, \( \phi_{31} \) value, and \( [\text{Fe/H}] \) for each star was plotted, using the spectroscopically-determined values for metallicity from Jurcsik in 1995 (see Figure 8).

Figure 8. Three-dimensional plot of the four data points from the 24-inch telescope, with a plane fit to the points.
A plane was then fit to the points using the Levenberg-Marquardt method of nonlinear least-squares. From Figure 8, the equation of this plane was found to be

\[ [\text{Fe/H}] = -5.473 - 5.603P + 1.476\phi_{31}. \]  

The reduced chi-square for this fit was 2.10. Compared to Equation (3), found by Jurcsik and Kovács, this equation is very similar, though this was done in the g’ filter instead of in V. These results do appear to be on the right track, however.

VIII. SDSS Data

The SDSS-II Supernova Survey, which takes images of a narrow band of sky along the celestial equator, also found about 300 RR Lyrae stars. About thirty relatively-complete light curves that also had metallicities from spectra were selected. These light curves were far less-populated than the stars for which data was taken at the 24-inch.

A template fitter was run on these light curves, and any bad points (stray data points that were far away from the template line) were removed from the light curve. Third-degree polynomial interpolation was then carried out on the light curves before a Fourier fit to order 15 was performed; of those, only the light curves whose Fourier fits were relatively-confined to the data points were kept. Figures 9 and 10 show examples of “good” and “bad” light curves.

Figure 9. Light curves from SDSS data with reasonably-good Fourier fits.
Figure 10. Light curves from SDSS data with poor Fourier fits. This occurs due to gaps in the light curve data.

Figure 11. Three-dimensional plot of the relatively-good SDSS light curves, with a plane fit to the points.

The final set of light curves was then plotted using metallicity, period, and $\phi_{31}$, similar to the process conducted for the 24-inch data. The metallicities here
were spectroscopically-determined by Lee in 2007. Figure 11 shows the resulting plot.

The equation for the plane that was fit to these points is shown in Figure 11, with a reduced chi-square value of 107.16. This equation is of similar magnitude and sign as Equation (4), found in Figure 8, though it is still significantly different and does not confirm the validity of the previous equation.

The plot of the data points shows a general trend in the expected direction, but unlike with the 24-inch data, there does not immediately appear to be a strong relationship. This is likely due to the lack of sufficient data for the SDSS light curves; even for the relatively-good light curves, there are still gaps in the data, and some are even missing vital components of their light curves, such as the bottom dip and ascending branch. The reduced chi-square value also suggests that more data points are needed for a believable fit.

IX. Conclusions and Further Research

The data taken at the 24-inch shows a clear relationship between metallicity and Fourier coefficients in the g’ filter. The Fourier coefficients are very sensitive to the completeness of the light curve, however, so the results using the SDSS data could be greatly improved if more data were acquired. About twenty points for each star are already available and just need to be processed. More data from the 24-inch would also be useful to fill out the gaps in the light curves of stars for which some data has already been taken.

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X. References

Lee et al. 2007, in preparation.